

Chapter 4

Magnetic Structure

INTRODUCTION

In this final chapter on the subject of structures, we turn attention to magnetic materials. Why? Because magnetic materials are illustrative of yet another level of structure that often arises in condensed matter beyond that of particle arrangements. As we will see, magnetic particles have a property of net spin and a magnetic moment whose orientation in space is largely unrestricted. Regardless of whether a large system of magnetic particles are positioned in an ordered or disordered manner, their spins represent an additional layer of ordering. The moments could be randomly oriented or aligned in a common direction. In ferromagnetic systems, these moments interact with one another to promote a local alignment of the moments that can eventually spread over the entire system. This is reminiscent of how pair-wise bonding between particles eventually leads to crystallization of a liquid and is the archetype for a wide variety of phase transitions in which order appears in the form correlated regions emerging from a disordered host.

THE ORDERING PROCESS

So far, our discussion of structure has focused entirely on *particles*: their relative positions and the forces that hold them together. We have seen that arrangements of particles fall into either an ordered or disordered pattern which can be characterized by the level of symmetry present. By virtue of its disorder, the liquid has rotational invariance and an infinite symmetry (on average). The crystal, however, conforms to a space lattice and possesses only a discrete set of symmetry operations. In the process of forming a crystal from the liquid, the symmetry is often said to be "broken".

{Insert Figure 4-1}

In addition to symmetry breaking, the transition from a liquid to crystal also induces a corresponding *sharpening* of the features found in the pair distribution function, $g(r)$. Again, this distribution function is a measure of the conditional probability that, given a particle is at the origin, another will be found a distance r away. In progressing from a gas to a liquid to a crystal as illustrated in Fig. 4-1, particle positions become increasingly correlated and the corresponding $g(r)$ evolves progressively sharper and distinct features due to the ever increasing precision in locating particle positions.

Correlations and Pattern Formation

Both symmetry breaking and correlation building are common elements in most all phase transitions and are nicely illustrated in the orientational ordering that occurs in

magnetic spin systems. But before we begin looking at magnetic materials, let us consider an example that may help to clarify how orientational structure discussed in this chapter is similar to, but also different from that discussed in previous chapters. Imagine a large country uniformly populated by people that are identical in every respect except for one property: their mood. For simplicity, we limit the possible moods to just that of either happy or sad. If these people go about their daily lives without interacting with one another, their individual mood would be a random result. There would be no bias for a person to be either happy or sad, and as illustrated in Fig. 4-2a, the country as a whole would appear uniformly complacent, with neither a balance of happy nor sad.

{Insert Figure 4-2}

But suppose that people do interact and can influence each other's mood. Experience suggests that a person surrounded by happy people tends to be happy while a person surrounded by sad people tends to be sad. Interactions with neighbors serve to bias a person's mood. With this biasing mechanism we might then expect to find patterns emerging within the society like those illustrated in Fig. 4-2b where small enclaves or communities of people develop with a common mood. Furthermore, we would expect to see these communities grow over time due to an inherent 'feedback' built into the biasing. The larger a community becomes, the more strongly it biases those potential converts on the periphery. This spontaneous growth would result in regions of correlated mood that coarsen and eventually merge into one another as illustrated in Fig. 4-2c.

But how do we characterize this growing pattern of correlated regions. A simple average of the property over the entire system would not necessarily reveal the

developing correlations taking place. As we saw in our example above, correlated regions may be substantial in size but could still be divided into nearly equal numbers of happy and sad communities. To illuminate the presence of these correlated patterns, we need to consider an appropriate *correlation function* of the form

$$C_X(\vec{r}_1, \vec{r}_2) = \langle X(\vec{r}_1)X(\vec{r}_2) \rangle, \quad (4.1)$$

where $X(\vec{r})$ represents the value of the property for a particle located at \vec{r} and the brackets again signify the required ensemble averaging. Conceptually, this correlation function is much like the pair distribution function discussed in Chapter 2. It expresses a conditional probability that given some central particle at \vec{r}_1 has a certain value of the property X , another particle at \vec{r}_2 will have the same value. In practice, the correlation function would be computed by selecting each particle in the system, in turn, as the central particle and sampling the similarity of those neighbors a distance $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$ away. Over distances shorter than a characteristic length, ξ , known as the *correlation length*, the property is similar and said to be correlated. At longer separation distances, the similarity vanishes and the property becomes uncorrelated.

Although our example involves people with mood swings, we could readily replace it with particles, each of which has some property whose value can match or differ from that of its neighbor. Magnetic particles with their intrinsic magnetic moment are one example.

MAGNETIC MATERIALS

When we speak of magnetic materials, we most often we think of materials like iron that can be either magnetized or de-magnetized by an external field and retain the final state of magnetization when the field is removed. Iron is a *ferromagnetic* material and, on a microscopic level, each atom of iron contains a miniature magnetic moment (a particle property) whose individual orientation is influenced not only by the external field that is applied, but is also biased by the orientation of its neighbors. Materials that possess a miniature moment, but which lack the neighboring interaction, are known as paramagnets. Although the moments of a paramagnet can be aligned with an applied field, paramagnets are unable to retain their magnetization when the field is removed.

In turn, we will look at both of these magnetic materials as well as consider the *diamagnetic* response seen mainly in non-magnetic materials that do not possess a permanent moment. We begin by inquiring into the origin of the miniature magnetic moment that endows magnetic materials with magnetization.

Magnetic Moments

To understand the origin of the magnetic moment of an atom, consider for a moment the simplistic Bohr model illustrated in Fig. 4-3 in which an electron orbits the nucleus with speed v on a circular path of radius ρ .

{Insert Figure 4-3}

The steady motion of the orbiting electron constitutes a continuously flowing loop of current,

$$I = \frac{\text{charge}}{\text{period}} = -e/(2\pi\rho/v) = -ev/2\pi\rho, \quad (4.2)$$

which together with the area of the loop produces a magnetic moment,

$$\mu_{orb} = IA = (-ev/2\pi\rho)\pi\rho^2 = -ev\rho/2. \quad (4.3)$$

Meanwhile, the orbiting mass of the electron produces an angular momentum, $L = mv\rho$, directed opposite the magnetic moment. Combining this with Eq. (4.3), we find that

$$\vec{\mu}_{orb} = -(e/2m)\vec{L}, \quad (4.4)$$

and conclude that the magnetic moment ultimately arises from the angular momentum of the electron. This conclusion is unchanged by a quantum mechanical approach, and since quantum mechanics requires the angular momentum be quantized in discrete units of \hbar , the orbital magnetic moment must appear in discrete units of a *Bohr magneton*,

$$\mu_B = e\hbar/2m = 9.27 \times 10^{-24} \text{ Am}^2.$$

In addition to the orbital angular momentum, there is also electron *spin* that contributes to the overall angular momentum of an atom. For the electron, spin is quantized and the quantum number can assume either $s = \pm\frac{1}{2}$. Without going into the detailed quantum mechanical treatment, the electron spin of a free electron is found to contribute to the magnetic moment by an amount,

$$\vec{\mu}_{spin} = -g(e/2m)\vec{S}, \quad (4.5)$$

where the so-called splitting factor, $g = 2.0023$.

For completeness, we should also acknowledge that there is yet another contribution to the magnetic moment that arises from the intrinsic spin of the nucleus. Again, the nuclear magnetic moment appears in quantized units, but of a smaller size,

$\mu_n = e\hbar/2m_p = 5.05 \times 10^{-27} \text{ Am}^2$, known as a *nuclear magneton*. Although this nuclear magnetic moment plays an important role in nuclear magnetic resonance (NMR), in magnetic materials it is completely masked by the much larger magnetic moment arising from the electron.

We now see that the magnetic moment has its origin in the combined orbital and spin angular momentum of the electron and we begin to get a glimpse as to why not all materials are magnetic. Consider any of the noble gas elements that form closed electron shells. Since each shell is filled, the total orbital angular momentum vanishes. Likewise, since each electron is paired to another with opposite spin (due to Pauli exclusion), the net spin angular momentum is also zero. Consequently, the noble gas elements have no net angular momentum and no permanent magnetic moment. They are non-magnetic. A similar fate is found for many elements of the periodic table in which electrons appear in pairs so that the vector sum of the angular momentum vanishes. Magnetic materials then develop only in those fortuitous instances when an outer electron shell is only partly filled and contains an odd number of electrons.

Determining the magnetic moment of an atom thus boils down to determining the total angular momentum of its constituent electrons and presents a complicated vector addition problem that is further compounded by rules of quantization. Luckily, this vector addition problem has already been tackled and is commonly treated using the Russell-Saunders scheme of spin-orbit coupling which can be found in most any introductory quantum mechanics textbook. In this scheme, the atom's total orbital angular momentum, $\vec{L}_{atom} = \sum_i \vec{L}_i$, and total spin angular momentum, $\vec{S}_{atom} = \sum_i \vec{S}_i$, are combined to obtain a total angular momentum,

$$\vec{J} = \vec{L}_{atom} + \vec{S}_{atom}. \quad (4.6)$$

The magnetic moments would be similarly added

$$\vec{\mu}_{atom} = \vec{\mu}_{orb}^{atom} + \vec{\mu}_{spin}^{atom} \approx -(e/2m)(\vec{L}_{atom} + 2\vec{S}_{atom}), \quad (4.6b)$$

producing a moment that is *not* aligned with \vec{J} . It is common to introduce an effective magnetic moment of the atom that is proportional to the total angular momentum and given by

$$\vec{\mu}_{atom} = -g'(e/2m)\vec{J}, \quad (4.7)$$

where (on can show Ex*) the splitting factor is now replaced by

$$g' = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}. \quad (4.8)$$

Diamagnetisism

We are keenly interested in those magnetic materials fortuitous enough to have a non-zero angular momentum. But, let us pause momentarily to consider how the remaining non-magnetic materials respond to an applied field. Although they lack a permanent moment, these non-magnetic materials do develop a weak magnetization that is *opposite* to the direction of the field. That is, they exhibit *diamagnetic* behavior as opposed to a *paramagnetic* response.

To understand the origin of the diamagnetic response, consider again the example of the single electron atom illustrated in Fig. 4-3. Suppose we apply a magnetic field, $\vec{B} = \mu_o\vec{H}$, directed upward, perpendicular to the orbital plane. This new field introduces an upward magnetic flux through the loop which, by virtue of Lenz' law (the one expressing Nature's displeasure with changing flux), causes the electron to react in such a

way as to produce a counter flux. Assuming that the orbital radius is fixed, a quick check of our right hand rule reveals that the needed counter flux is achieved by having the electron speed up (or slow down, if the electron were orbiting in the opposite direction). One can show (Ex. 2) that the electron suffers a change in its angular velocity by an amount,

$$\Delta\omega = eB/2m, \quad (4.9)$$

known as the *Larmor* frequency. This speed change is common to all electrons in the orbital and, for an atom with Z electrons, the applied magnetic field thus induces a net change in the current by an amount,

$$I_{induced} = -Ze(\Delta\omega/2\pi) = -Ze^2B/4\pi m. \quad (4.10)$$

This current change in turn induces an average magnetic moment,

$$\langle \mu_{induced} \rangle = -(Ze^2B/4m)\langle \rho^2 \rangle \quad (4.11)$$

directed opposite the applied magnetic field. Collectively, this produces a net magnetization per unit volume of

$$\vec{M} = n\langle \vec{\mu} \rangle. \quad (4.12)$$

where n is the particle number density. Because diamagnetic materials respond to an applied field by producing an induced magnetic moment proportional to the field but opposite in direction, the magnetic susceptibility of a diamagnet,

$$\chi_m = \frac{dM}{dH} = n\mu_o \frac{\langle \mu_{induced} \rangle}{B} = -(Ze^2n\mu_o/4m)\langle \rho^2 \rangle, \quad (4.13)$$

is always less than zero. While the diamagnetic response is only measureable in non-magnetic materials, it is nevertheless found universally in all materials including those magnetic materials that have a permanent magnetic moment due to a net angular

momentum. The size of the induced moment is however much smaller than that of the permanent moments and in magnetic materials the diamagnetic contribution is masked by the paramagnetic response. Values for the susceptibility for a handful of common elements are listed in Table 4-1.

Table 4-1 Susceptibility of several elements at standard temperature and pressure (values obtained from *Handbook of Chemistry and Physics* (1983)).

Diamagnetic		Paramagnetic	
Element	Susceptibility	Element	Susceptibility
Bismuth	-1.6×10^{-4}	Sodium	8.5×10^{-6}
Gold	-3.4×10^{-5}	Aluminum	2.1×10^{-5}
Silver	-2.4×10^{-5}	Tungsten	7.8×10^{-5}
Copper	-9.7×10^{-6}	Platinum	2.8×10^{-4}
Hydrogen	-2.2×10^{-9}	Gadolinium	4.8×10^{-1}

Paramagnetism

As we learned earlier, magnetic materials are those whose constituent atoms possess a net, non-zero, magnetic moment in the absence of any applied magnetic field. Such materials are generally referred to as paramagnetic because the effect of an applied field produces a torque that tends to align moments in the direction of the field with a corresponding lowering of internal energy given by,

$$U = -\vec{\mu} \cdot \vec{B}. \quad (4.14)$$

In difference to ferromagnetic materials, which we will discuss in the next section, this alignment of moments with the field disintegrates at any finite temperature when the field is removed due to incessant thermal agitation. Consequently, the magnetization vanishes as the moments return to a disordered pattern of random orientations.

Let us consider then a paramagnetic material with total angular momentum quantum number j . How does the magnetization of this material depend on both the applied field (acting to align the moments) and the temperature (acting to randomize the orientations)? The answer to this question requires a thermodynamic approach. Imagine as illustrated in Fig. 4-4 that the field is applied along the z -axis. Because angular momentum is quantized, its component along the z -direction is also restricted to values $J_z = m_j \hbar$, where the magnetic quantum number, m_j , ranges from $m_j = j, (j-1), \dots, 0, \dots, -(j-1), -j$. From Eq. (4.7), this then implies that the component of $\vec{\mu}_i$ along the field is similarly quantized as $\mu_z = -g' \mu_B m_j$.

{Insert Figure 4-4} use lower case j's

Since we also anticipate no net magnetization to appear in either the x - or y -directions, the magnetization we seek is given by Eq. (4.12) using only the average value of μ_z consistent with conditions of thermodynamic equilibrium. Using Boltzmann statistics, this average can be expressed as

$$\langle \mu_z \rangle = \frac{\sum_{-j}^j (-g' \mu_B m_j) e^{-U_j/k_B T}}{\sum_{-j}^j e^{-U_j/k_B T}}, \quad (4.15)$$

where

$$U_j = -\mu_z B = -(-g' \mu_B m_j) B = m_j (g' \mu_B B). \quad (4.16)$$

One can show (Ex. 5) that this relation reduces to

$$\langle \mu_z \rangle = (g' \mu_B j) B_j(y), \quad (4.17)$$

where $y = g' \mu_B j B / k_B T$, and

$$B_j(y) = \left\{ \left(\frac{2j+1}{2j} \right) \coth \left[\frac{(2j+1)y}{2j} \right] - \left(\frac{1}{2j} \right) \coth \left(\frac{y}{2j} \right) \right\}, \quad (4.18)$$

is known as the *Brillouin* function.

{Insert Figure 4-5}

Good agreement of Eq. (4.17) with experiment is demonstrated in Fig. 4-5 for several materials of differing j . In all instances, the magnetization increases with increasing field reaching a point of saturation in which all the moments are aligned with the field. In the weak field regime, the magnetization is roughly proportional to the applied field. Here, we can approximate the Brillouin function for small argument as

$$\lim_{y \ll 1} B_j(y) = \frac{y(j+1)}{3j}, \quad (4.19)$$

and obtain the susceptibility as

$$\chi_m = \frac{dM}{dH} = n \mu_o \frac{g'^2 \mu_B^2 j(j+1)}{3k_B T} = \frac{C}{T}, \quad (4.20)$$

a result known as the *Curie law*.

An example of paramagnets exhibiting Curie behavior are shown in Fig. 4-6. In paramagnets, there is no interaction between neighboring moments and the ability to

align moments using an external applied field is countered only by the thermal agitation present at finite temperatures. When this thermal agitation vanishes at absolute zero, the alignment of magnetic moments then occurs without competition and, as the diverging susceptibility suggests, any tiny field will be capable of aligning all the moments.

{Insert Figure 4-6}

Ferromagnetism

Unlike paramagnets, ferromagnets are able to retain their magnetization when the external field is removed. In actuality, this is only true if they are held at a temperature below the so-called Curie temperature, T_c . Above the Curie temperature, the material responds like a usual paramagnet and the magnetization vanishes when the field is removed.

To interpret the ferromagnetic behavior it is somewhat natural to suppose that there exists in these materials some internal interaction between magnetic moments that biases neighboring moments to adopt a common orientation. Indeed, early theories of the ferromagnetic state advanced by Pierre Weiss fashioned this interaction in the form of an internal magnetic field, self-generated by the net alignment of moments and thus proportional to the net magnetization,

$$\vec{H}_{\text{int}} = \lambda \vec{M} \text{ or } \vec{B}_{\text{int}} = \mu_o \lambda \vec{M}. \quad (4.21)$$

This so-called *molecular field* was seen as being both a result of the aligned moments and yet also a stimulus for additional alignment. Inherent in this dual nature is a sort of

feedback mechanism that can promote the rapid growth of regions of correlated magnetic moment below the Curie temperature.

Since the molecular field is proportional to the magnetization, the magnetization can then be expressed as a modified form of the Curie law,

$$M = (C/T)(H + H_{\text{int}}) = (C/T)(H + \lambda M), \quad (4.22)$$

where the molecular field is merely adding to that which is applied externally. Evident in this modified expression is the feedback alluded to earlier: the applied field promotes an incipient magnetization which in turn increases the effective field to promote even more magnetization. We can rearrange Eq. (4.22) to obtain

$$M = \frac{C}{(T - C\lambda)} H, \quad (4.23)$$

and obtain the susceptibility as

$$\chi_m = \frac{C}{(T - T_c)}, \quad (4.24)$$

where $T_c = C\lambda$ is identified with the *Curie temperature*. This temperature variation for ferromagnetic materials is known as the Curie-Weiss law and is illustrated in Fig. 4-7. In difference to a paramagnetic material whose susceptibility diverges only at absolute zero, the susceptibility of a system of interacting magnetic moments diverges at the finite Curie temperature. It is at this temperature where the molecular field, acting to align moments, achieves dominion over the thermal agitation working to disorder the moments.

{Insert Figure 4-7}

Exchange Interaction

Prior to the development of quantum mechanics, the only field known to exist within a magnetized specimen was a dipolar field collectively produced by the aligned magnetic moments themselves. At the location of each moment, this dipolar field has a magnitude given roughly as

$$B_{dip} \approx \frac{\mu_o}{4\pi} \left(\frac{2\langle \mu \rangle}{a^3} \right), \quad (4.25)$$

where a is the spacing between particles. Unfortunately, this dipolar field is too weak to account for most ferromagnetic materials, including iron whose Curie temperature ranges near 1000 K. Combining Eq. (4.20) and Eq. (4.25), we can express the Curie temperature for the case of a dipolar field as

$$T_c = C\lambda \approx \left[\mu_o \frac{g'^2 \mu_B^2 j(j+1)}{3k_B} \right] \left(\frac{1}{2\pi a^3} \right). \quad (4.26)$$

For iron (see Ex. 3) this yields only about 3 K, which is some two and a half orders of magnitude smaller than the experimental result.

The resolution to this problem came eventually from quantum mechanics. In a fashion analogous to our discussion of the covalent bond in Chapter 3, a quantum mechanical treatment of the overlapping wavefunction of two neighboring atoms produces an addition contribution to the potential energy of the form

$$u(r_{ij}) = -2J_{ex}(r_{ij}) \vec{S}_i \cdot \vec{S}_j, \quad (4.27)$$

where \vec{S}_i and \vec{S}_j are the respective spins of the two atoms. This interaction is known as the *exchange interaction*, and the energy parameter J_{ex} , whose dependence on the atom separation is illustrated in Fig. 4-8, is often referred to as the exchange integral. As seen in the figure, for a range of intermediate atomic separations, the exchange integral is positive and results in a lowering of internal energy if the two neighboring spins are

aligned. This is the source of the mysterious biasing of moments that stabilizes the magnetization in a ferromagnetic material. Although the source is not really a molecular field but rather a pairwise interaction between adjacent spins, it is nevertheless convenient to treat the interaction as if it were the result of mean field as described by Eq. (4.21).

{Insert Figure 4-8}

Correlated Domains

Now that we understand the origins of ferromagnetic behavior, we can finally return to our original theme concerning orientational pattern formation in magnetic materials by considering what happens when we cool a ferromagnet toward its T_c in the absence of any field. Far above T_c , the moments are orientated randomly because the thermal energy far exceeds that of the exchange interaction and serves to thwart its biasing effect. As we cool, we would expect to find small communities of similar spin orientation developing, as illustrated in Fig. 4-9.

{Insert Figure 4-9}

The specific orientation of a region is random and reflects only that orientation which happened to be dominant when the region first began to form. To characterize this developing pattern, we would naturally introduce a correlation function, the *moment-moment correlation function*

$$\Gamma(\vec{r}) = \langle \vec{\mu}(0) \cdot \vec{\mu}(\vec{r}) \rangle, \quad (4.27)$$

which would be zero at very high temperatures (reflecting the random orientations) but would develop a finite, non-zero value over an extent characterized by a correlation length $\xi \gg a$ at lower temperatures. Arriving at T_c , we would observe the correlation span over great distances, comparable to the size of the system itself. The correlated regions would develop into so-called *domains* with a common orientation of the magnetic moment.

Hysteresis Loop

What happens if we now apply an external field to this zero-field cooled specimen? Our magnetization would follow a path illustrated in the Fig. 4-10, known as a *hysteresis* loop. Our specimen starts off in an unmagnetized state. Although there exist large domains of uniformly magnetized regions, these domains are individually oriented in random directions resulting in little or no initial magnetization. As we apply a field, those domains with orientation in the field direction begin to grow at the expense of other domains. These favored domains expand their boundaries by converting spins near the periphery of the domain. Consequently, the ill-favored domains shrink away. At some very large applied field, the favored domains have grown to macroscopic proportions and the magnetization saturates. As the field is then removed, the new domain structure adjusts slightly but the favored domains remain dominant due to the internal biasing provided by the exchange interaction. When the field is completely removed, the finite magnetization that remains is known as the *remanence*, M_R .

{Insert Figure 4-10}

In order to de-magnetize the specimen, a reverse field must be applied. In applying this reverse field, the dominant domains are now shrunken while domains of opposite orientation (those aligned with the reversed field) begin to grow. Eventually, a situation is reached at a field of magnitude H_c known as the coercive field (or *coercivity*) where the net magnetization vanishes. Further increase of the reverse field beyond H_c drives the system again to a state of saturation.

SUMMARY

- Magnetic behavior appears in atoms with a net total (orbital plus spin) angular momentum, \vec{J} .
- The permanent magnetic moment of an atom is proportional to J and appears in units of a Bohr *magneton*, $\mu_B = e\hbar / 2m$.
- All materials exhibit a diamagnetic response. For non-magnetic materials, the response is observed as a weak magnetization opposite the applied field.
- Paramagnets lack any interaction between neighboring moments and lose their magnetization whenever a field is removed. Ferromagnets experience an interaction between neighboring moments and can retain their magnetization if the field is removed at temperatures below the *Curie temperature*, T_c .
- The interaction between moments in a ferromagnet arises from a quantum mechanical interaction between neighboring atomic spins, $u(r_{ij}) = -2J_{ex}(r_{ij})\vec{S}_i \cdot \vec{S}_j$, known as the *exchange interaction*.
- Near T_c , ferromagnets develop regions of correlated moments which increase in size as T_c is approached.

EXERCISES

4-1. A power transformer consists of two windings around a common ferromagnetic core and is used in ac circuits to step up (or down) an ac voltage. During each cycle, the ferromagnetic core repeats the process of growing and shrinking domains. (a) Show that the work done in one cycle of a hysteresis loop equals the area enclosed by the loop. (b) Which would be better for a power transformer: a core with a large coercivity or a small coercivity? Why? (Hint: you might make use of the following thermodynamic relation: $dW = HdM$)

4-2. Consider the centripetal force on the orbiting electron of the classical Bohr model illustrated in Fig. 2. Consider both a situation with and without a magnetic field present and show that when the field is present and directed upward (perpendicular to the orbital plane), the electron speeds up by $\Delta v \approx \rho e B / 2m$ and that the change in its angular velocity then equals the Larmor frequency given in Eq. (4.9). As a follow up to this, determine how large of magnetic field is needed to cause the 1% change in the orbital speed.

4-3. In the text, the T_c for iron was estimated to be only about 3 K based on a classical molecular field due to the dipolar interaction. Verify this by direct substitution into Eq. (4.26) using for iron the angular momentum $j = s = 1$ and a particle spacing $a = 1 \text{ \AA}$.

4-4. Compute the diamagnetic susceptibility for a gas of hydrogen atoms (at standard temperature and pressure). Recall that the ground state wave function for a hydrogen

atom is $\psi(r) = (\pi a^3)^{-1/2} e^{-r/a}$, where $a = \hbar^2 / me^2 = 0.529 \text{ \AA}$. Note also that

$\langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle = \frac{2}{3} \langle r^2 \rangle$ for a spherically symmetric orbital. Compare your result with

the experimental value listed in Table 4-1.

4-5. Obtain the form of the Brillouin function given in Eq. (4.18) starting from Eq.

(4.15). As way of hint, notice that:
$$\frac{\sum m e^{mx}}{\sum e^{mx}} = \frac{d}{dx} \left\{ \ln \left(\sum e^{mx} \right) \right\}.$$

4-6. Consider the simple two state paramagnet with $j = s = 1/2$. Show that for such a paramagnet, the magnetization is given simply as: $M = n\mu_B \tanh(\mu_B B / k_B T)$. Using this result, determine the magnitude of magnetic field required to produce a magnetization of half its saturation value at (a) 300 K and (b) 1 K.

4.7. Draw an arbitrary $\vec{L}_{atom} + \vec{S}_{atom}$ vector combination that results in \vec{J} . On this same drawing include also the magnetic moment addition indicated by Eq. 4.6b that would

accompany your vector choices. Given that $|\vec{L}_{atom}| = \sqrt{l(l+1)}\hbar$ and $|\vec{S}_{atom}| = \sqrt{s(s+1)}\hbar$,

show that the projection of $\vec{\mu}_{atom}$ onto the \vec{J} direction, $\frac{\vec{\mu}_{atom} \cdot \vec{J}}{|\vec{J}|}$, equals the result given in

Eq. 4.7 with the splitting factor of Eq. 4.8.

Suggested Reading:

Chaikin and Lubensky provide additional depth on the subject of correlations, but at an advanced level that might not appeal to all. For a good introduction to magnetic materials I recommend reading the sixth chapter of Griffiths.

Chaikin and Lubensky provide additional depth on the subject of correlations, but at a higher level that might not appeal to all.

D. J. Griffiths, *Introduction to Electrodynamics*, 3rd Ed. (Prentice Hall, New Jersey, 1999).

P. M. Chaikin and T. C. Lubensky, *Principles of Condensed Matter Physics* (Cambridge University Press, New York, 2003).

C. Kittel, *Introduction to Solid State Physics*, 8th Ed. (John Wiley and Sons, 2005).