Creighton University — Professor G. Duda

Physics 531: Quantum Mechanics — Fall 2013

Project #1: Quantum Mechanical Tunneling and Alpha Decay

To set the stage for this project, please make sure youve watched and worked through the powerpoint lecture on Blueline (in the modules section under Project #1) titled Spectroscopy, the Atom, and Early Nuclear Physics.

One of the first mysteries of quantum mechanics was the quantization of energy, which manifested itself as discrete absorption and emission lines from molecules and atoms. Although we have not yet tackled the hydrogen atom, we saw (or you will see) how energy quantization arises in the infinite and finite wells: boundary conditions force particles to behave as standing waves leading to fixed modes and quantized energies. Your project is to take what we have learned in solving the 1D Schrodinger Equation and to apply it to quantum tunneling, the strange phenomenon in quantum mechanics where particles can "tunnel" through potential barriers even though they do not have sufficient energy classically. Quantum tunneling is an amazingly important physical phenomenon. In fact, much of our modern technology like flash drives and Josephson Junctions depend on quantum tunneling for their operation. The Scanning Tunneling Microscope (STM) which has allowed physicists to discern and manipulate individual atoms on surfaces operates because of quantum tunneling. And it turns out quantum mechanical tunneling is even important in stellar fusion and in how the smell receptors in our noses function. This project will help you learn about quantum tunneling in an important application in nuclear physics: the alpha decay of uranium.

Project in a Nutshell: In this project you will solve a great puzzle of the late 1920s in nuclear physics: the alpha decay of Uranium. The puzzle is essentially the following: the how can alpha particles created in the interior of a nucleus escape despite having insufficient energy to surmount the Coulomb barrier of the nucleus? The answer is quantum mechanical tunneling, and you will perform the calculations that demonstrate the validity of this supposition. Furthermore, you will use your calculations to determine the half-lives of Uranium and selected other even-even nuclei such as Polonium, Bismuth, Thorium, and Radium given the energies of their emitted alpha particles.

0 Project Report

Each team will submit a detailed, professional quality, literature style report of your application of quantum mechanics to alpha decay. This report will cover what you did, why you did, what you learned, and will comment on how effective our model replicates experimental results for this system.

- There is no page limit, but you should try to maintain clarity and succinctness in your report (I would recommend about five single-spaced pages).
- Model your report after a paper in the professional physics literature: include a title, list of authors, and a one-paragraph abstract that summarizes the aim, scope, results, and conclusions of your project.
- You should use the LaTeXtemplate on Blueline2 to format your paper in a two-column, Phys. Rev. D. format.
- Include high quality figures, equations, and tables that support your analyses and conclusions.
- Cite information sources as appropriate (you must include a bibliography). You will never find a scientific paper in the literature without citations! Even the greats like Feynman, Landau, and others cited others work.
- The intended target audience for your research papers is your fellow students in this course. As you are an advanced, knowledgeable, and intelligent group, I expect a high level of detail and mathematical sophistication in your projects; however, such detail should be at a level at which your fellow classmates can understand the material.
- The handout "Guidelines for Scientific Writing" by Eric. D'hoker at UCLA should be a useful resource - this is posted on Blueline2.

Your report will be assessed according to the following criteria. Additional details on these competency assessments are provided in the Project Grading Rubric (available on Blueline2).

1. Qualitative analysis Are your arguments clear? Are you able to use physical data and researched information to explain technical phenomena and support your conclusions? Do you make appropriate connections among the various technical concepts and information, and between the technical and contextual information? Do you support your data analyses and conclusions with quantum theory and nuclear physics? Do you explain discrepancies? Do you make good use of estimation (if appropriate)?

2. Quantitative analysis Are your calculated data accurate? Do you make appropriate use of published information and theory to support your data and quantitative analyses? Do your quantitative results connect to and support your qualitative discussion? Do you present your calculations in an organized and logical manner?

3. Communication Do you make effective use of written communication in the report? Is your report well-organized, well-written, and appropriate for the audience? Are the arguments and goals clear, and does the report support these arguments and goals? Are the mechanics (spelling, grammar, word choice, punctuation) well-executed? Do you make logical and well-supported arguments? Do you make good use of graphs and images? Is reference information and evidence carefully woven into the text? Does your report follow the model of a paper in the physics literature?

Detailed grading rubrics for the project report are posted on Blueline2. You are encouraged to read these rubrics as you write up your project, and to communicate with the instructor if there are areas of uncertainty. You will use these rubrics for your competency self-assessment.

In-class teaming reflections and a brief teaming survey will be used at the end of the project to help maintain effective teams. The results from this survey will not affect your grade for this first project; however, participation in the reflection and feedback process will affect your grade. The reflections and surveys administered at the end of the project are simply intended to spark conversations among teammates and help you recognize areas of strength and areas potentially in need of further development. Individual team member contributions and behaviors will be evaluated at the end of the project using the Comprehensive Assessment of Team Member Effectiveness (CATME) survey.

1 General Background: Nuclear Physics at the Dawn of the 20th Century - Stage 1

Well be tackling our projects in stages. Each stage will suggest questions to consider or strategies to employ or calculations to perform that need to be completed before moving on to the next stage. Some of this is scaffolding and will be reduced in future projects, and some of it is very much a quality control issue. Since you are not sitting through lectures, I very much need to determine that youve thought through and worked through the material at the appropriate level. But most of all, the breaking of the project up into stages is meant to help you - this is a very ambitious calculation we're attempting, and setting you loose on the project without some guidance wouldn't result in the best learning experience. This first stage sets up a bit of the background and history of this particular project, and all you need to do is read this carefully.

1.1 Radioactivity

For a review of the history of the discover of radioactivity, I highly recommend that you read the handout titled "The History of the Discovery of Radiation and Radioactivity" which I have posted to Blueline2 in the Project #1 module folder. That said, let's review a few key details which will be relevant to this project.

Between 1899 and 1901 Rutherford along Paul Villard studied radioactivity and classified radiation based on its penetration powers into three types: alpha, beta, and gamma, where alpha is the least penetrating and gamma the most penetrating. At first, the identity of the alpha, beta, and gamma radiation were unknown. However, in 1900 Becquerel, using J.J. Thompson's method of measuring the e/m ratio, showed that beta particles are in fact electrons. In1907 Rutherford and Thomas Royds showed conclusively that alpha particles were doubly-ionized Helium nuclei, and William Henry Bragged showed that gamma rays were electromagnetic radiation (mainly due it's non-deflection by a magnetic field - Rutherford and Edward Andrade would later measure the wavelength of gamma rays). These three types of radiation would prove powerful tools in investigating and understanding the world of the atom.

1.2 Plum Pudding and the Atom

In the early part of the first decade of the 20th century, the dominant model of the atom was that of J.J. Thompson: an atom is like a plum pudding in that the positive of the atom is smeared throughout a mostly uniform spherical volume with electrons embedded like raisins. However, the famous gold-foil experiment conducted by Geiger and Marsden in 1909 under the direction of Ernest Rutherford shattered this paradigm. While observing the elastic scattering of alpha particles (helium nuclei) from radium bromide from a thin piece of gold foil only several atoms thick they noticed that although most alpha particles were not substantially deflected, some 1in 8000 were deflected by angles larger than 90°. Rutherford is reported to have said,

> “It was quite the most incredible event that has ever happened to me in my life. It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you.”

From this observation Rutherford concluded that the plum-pudding model was quite incorrect, and rather that the majority of the mass of the atom and all of the positive charge was concentrated in a small, compact object we now call the nucleus. From the frequency of these large scattering events Rutherford was able to show that the nucleus must be smaller than 10^{-14} m in radius.

1.3 The Mystery of Alpha Decay

Now that the structure of the atom was better understood, scientists turned to the question of how α-particles were emitted from radioactive substances. Rutherford was said to be especially bothered by this question. Here’s why. Rutherford assumed that potential in the vicinity of a nucleus could be separated into two parts: i) a non-Coulombic part which was attractive and responsible for holding the nucleus together and which operated in the interior of the nucleus over distances $< 10^{-14}$ m, and ii) an electrostatic-repulsion (Coulombic-part) potential outside the interior of the nucleus. The two potentials should match up at r_0, the “rim” of the nucleus. Rutherford himself conducted scattering experiments in which he sought to probe the Coulombic part of the potential. Using alpha particles from radioactive Thorium C’ (which in modern notation is ^{218}Po), he probed the Coulombic potential of Uranium, and found that the Coulomb barrier was at least as high as 8.57 MeV. However, the alpha particles that Uranium decay emitted (through he reaction $^{238}\text{U} \rightarrow {}^{234}\text{Th} + {}^{4}\text{He}$) only had about 4.2 MeV of kinetic energy. Hence the puzzle. How were alpha particles escaping from the interior of the Uranium nucleus when they had insufficient kinetic energy to surmount the Coulomb barrier?

Many torturous theories were invented to try and explain this phenomenon. Rutherford personally postulated that perhaps an alpha particle in the interior of the nucleus combined with two electrons, became a neutral object, and hence could escape through the potential wall. In the process the electrons were left behind. However, it is not hard to see the flaws in this proposal: as one physicist put it, “this assumption appears to be quite unnatural and hardly corresponds to the actual facts.”

It was at this point that George Gamow entered the story. Gamow, a Russian-Ukranian physicist born Georgiy Antonovich Gamov in Odessa, was finishing his Ph.D. and had recently moved to Gottingen to work on quantum theory. Unlike most of his contemporaries, he shied away from crowded field of further developing quantum mechanics in the atomic and molecular realm, and instead was searching for a new field to contribute to. He felt quantum mechanics was already too mathematically sophisticated and crowded for his taste. Gamow began searching through the literature for an interesting problem to work on. According to his autobiography, Gamow happened across Rutherford's 1927 paper "Structure of the Radioactive Atom and the Origin of the α-Rays" in which Rutherford discussed the mystery of alpha-decay. Gamow said, "before I closed the magazine I knew what *actually* happened in this case ...".

What was Gamow's insight into the problem? Quantum tunneling. Gamow quoted from the papers of Nordheim and Oppenheimer on wave mechanics: "In wave mechanics there always exists a transition probability different from zero for a particle to get from one region to another which is separated from the first one by an arbitrary, but finitely high, potential barrier." Gamow's solution to the problem, was that the puzzle of alpha decay was not a problem at all. The alpha particle did not need to have sufficient energy to overcome the Coulombic barrier of the nucleus - it simply tunneled through it.

2 The Rectangular Barrier - Stage 2

Gamow began his calculation of the alpha decay of Uranium with a simplified model. He looked at the quantum mechanical probability for an alpha particle of energy E to tunnel through a square barrier of height U_0 and width l.

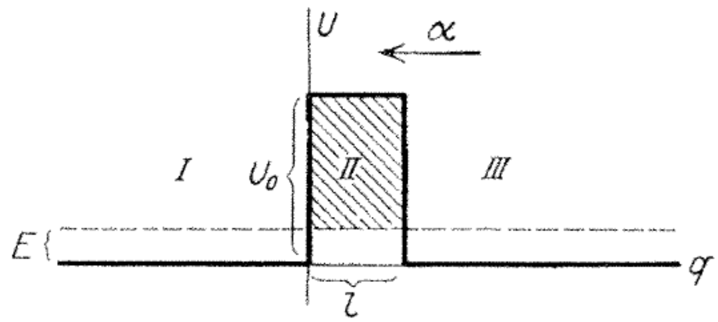

Figure 1: Gamow's sketch of α-particles of energy E incident from the right on a square barrier of height U_0 and width l. Gamow uses q instead of x, i.e. our wave function is $\psi(q)$.

Steps/Tasks you should complete in this stage of the project:

1. Using your knowledge of solutions to the 1D Schrodinger Equation for potential wells, write down the solution to the TISE for Regions I, II, and III. Make sure you include both incoming and reflected particles in Region III but only transmitted particles in Region I.

2. Apply the appropriate boundary conditions to your wave functions.

3. Using the machinery of 1D scattering find the exact transmission probability that the alpha particle will tunnel through the barrier.

4. In the case where the barrier is both high and broad, show that the transmission probability depends essentially on the exponential factor

$$T \sim \exp\left[-2\sqrt{\frac{2m}{\hbar}(U_0 - E)}l\right]$$

3 Refining the Model - Stage 3

The model we worked with in Stage 1 was of course not very realistic. One issue we need to deal with is that probability (and hence particle number) is conserved in non-relativistic quantum mechanics, whereas in particle decay conservation of particle number is of course violated. Since we're dealing with Uranium decay $^{238}\mathrm{U} \to {}^{234}\mathrm{Th} + {}^{4}\mathrm{He}$, we need to learn to deal with violations of probability conservation. In fact, Gamow's refined model for alpha decay does just this.

In Quantum Mechanics the continuity equation reads

$$\frac{\partial P(x,t)}{\partial t} + \frac{\partial}{\partial x} j(x,t) = 0,$$

where $P(x,t)$ is the probability ($\psi^*\psi$) and $j(x,t)$ is the probability current given by

$$j(x,t) = \frac{\hbar}{2mi}\left(\psi^* \frac{\partial \psi}{\partial x} - \psi \frac{\partial \psi^*}{\partial x}\right).$$

The continuity equation is simply a statement of conservation of probability (the change in probability is given by the flow of probability into or out of the region in question).

5. Derive the continuity equation for the Time Dependent Schrodinger Equation by multiplying the S.E. by ψ^* from the left and the complex conjugate of the S.E. by ψ from the right and subtracting the two. Identify $P(x,t)$ and $j(x,t)$ as defined above.

6. Suppose we add a complex term to our potential in the 1D Schodinger Equation, i..e

$$i\hbar \frac{\partial \psi(x,t)}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\psi(x,t) + \left(V_1(x) + iV_2(x)\right)\psi(x,t).$$

   Show that if the potential (or the energy) is complex that probability is not conserved. Give an expression for the rate at which probability is lost or gained. Explain how we might be able to use this effect to model alpha decay.

For a slightly more realistic model of alpha decay, Gamow next considered a double well structure. His idea was to look for stationary solutions of the Schrodinger Equation which represented a current of particles which were outgoing from a central region.

7. Can stationary solutions to the Schrodinger Equation represent a flow of particles out from a central region? What does your earlier examination of the continuity equation tell you?

Here's a sketch of Gamow's new model:

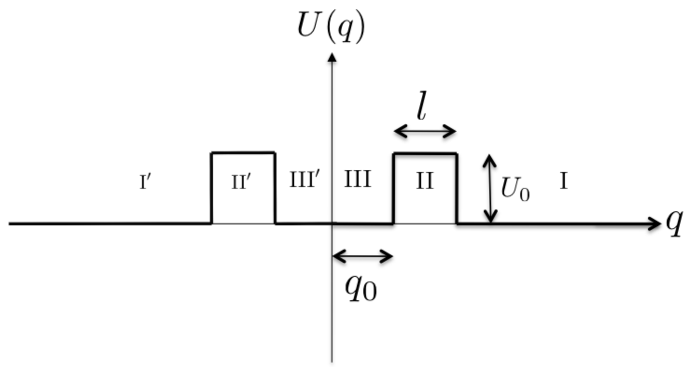

Figure 2: Gamow's new model of two symmetrical rectangular potential barriers. The barriers are separated by a distance $2q_0$ and each is a width l. The Roman numerals represent the five different regions of the potential.

8. Show that if we want the solutions in the regions I $q < -(q_0 + l)$ and Region I′ $q < -(q_0 + l)$ to represent particles coming out from the central region, the wave functions should be

$$\psi(q) = Ae^{i(Et/\hbar - qk' + \alpha)} \qquad \text{and} \qquad \psi(q) = Ae^{i(Et/\hbar + qk' + \alpha)}$$

respectively where $k' = \sqrt{2mE/\hbar^2}$. Explain why the constant α is the same for both regions.

9. Show that our choice our wave functions in Regions I and I' violate the continuity equation, i.e. there is a net outflow of probability from the central well between the two square barriers.

10. Write down solutions to the time independent Schrodinger Equation for Regions II, II′, III, and III′.

11. What are the boundary conditions you must satisfy?

12. Determine constants in regions II, II′, III, and III′ in terms of A and α. Are you able to fully satisfy the boundary conditions? Explain.

4 Complex Energy Solutions - Stage 4

Gamow realized that due to the large half-life of Uranium, the decay constant this implied was small in comparison to nuclear energies. This in turns means that the current we found as a result of writing the solutions in Regions I and I′ as outgoing plane waves is also small. What this means is that the violation of the continuity equation is small and we can essentially treat the alpha particle state inside the barrier (inside $q < (q_0 + l)$ and $q > -(q_0 + l)$) as nearly stationary. To get everything to work out we finally make use of one of the chief results you derived in Stage #2 - a complex energy (or potential) leads to a violation of the continuity equation. Here we'll try to find the decay constant by quantifying the amount of violation of the continuity equation.

Therefore, we let

$$E = E_0 + i\frac{\hbar\lambda}{2},$$

where λ is the decay constant (note that $\lambda\hbar$ has units of energy) and E_0 is the usual alpha particle energy.

12. Show that $\hbar\lambda$ is small compared to E_0 by looking up the decay constant for ^{238}U.

Because $\lambda\hbar$ is so small, we can keep the same solutions to the Schrodinger Equation as we had earlier, but modify them simply by multiplying each solution by $e^{-\lambda t/2}$.

13. By starting with the continuity equation, i.e.

$$\frac{\partial}{\partial t}\int_{-(q_0+l)}^{+(q_0+l)} \psi^\star \psi dq = -2\frac{\partial}{\partial q} j_I(q),$$

   show that we can can find an expression for λ by calculating

$$\frac{\partial}{\partial t} e^{-\lambda t}\int_{-(q_0+l)}^{+(q_0+l)} \psi^\star \psi dq = -2\frac{A^2\hbar}{2mi} 2ik' e^{-\lambda t}$$

14. Using your calculations from Step #11, perform the integral above and show that λ may be written as

$$\lambda = \frac{8\hbar k' \sin^2\theta}{m\left[1+\left(\frac{k}{k'}\right)^2\right] 2(l+q_0)\kappa} \cdot e^{-2l\sqrt{2m(U_0-E)/\hbar^2}},$$

where κ is a constant of order one and $\sin\theta = \left[1+(k'/k)^2\right]^{1/2}$.

Note that with our simple model we've derived an expression for how the decay constant of Uranium should depend on the energy of the emitted alpha particle. However, this is still highly dependent on the size of the potential well (q_0 and l) as well as the height of the potential barrier U_0.

5 The WKB Approximation - Stage 5

Gamow next took advantage of the WKB Approximation (named for physicists Wentzel, Kramers, and Brillouin) to find the transmission probability for alpha decay. The basic idea of the WKB approximation is the following. The time independent Schrodinger Equation can be written as

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi = E\psi.$$

This can be re-written as

$$\frac{d^2\psi}{dx^2} = -\frac{p^2}{\hbar^2}\psi,$$

where $p(x) = \sqrt{2m(E - V(x))}$, which is the classical formula for the momentum of a particle of energy E moving in a potential $V(x)$. We can write the solution to the Scrodinger Equation as

$$\psi(x) = A(x)e^{i\phi(x)},$$

where $A(x)$ is a position dependent amplitude and $\phi(x)$ is a position dependent phase.

15. Show that when $A(x)$ varies slowly (so that $A''(x)$, i.e. the second derivative of A is negligible) that $\psi(x)$ can be written as

$$\psi(x) \simeq \frac{C}{\sqrt{p(x)}}e^{\pm i/\hbar \int p(x)dx},$$

where C is a constant.

16. Now why are we looking at the WKB Approximation? Well, it turns out to be extraordinarily useful for tunneling when the barrier is high and/or wide (and our $E < V$). Let's set this up. Suppose we have a barrier between $x = 0$ and $x = a$ of essentially indeterminate shape for now. To the left of the barrier we write our solutions as

$$\psi(x) = Ae^{ikx} + Be^{-ikx},$$

where these two terms represent incoming and reflected waves respectively. To the right of the barrier $x > a$ we have

$$\psi(x) = Fe^{ikx},$$

which represents the transmitted wave. Show that the wave function inside the potential barrier by the WKB approximation can be written as

$$\psi(x) \simeq \frac{C}{\sqrt{|p(x)|}} e^{-1/\hbar \int_0^x |p(x')|dx'},$$

where again C is a constant.

17. Essentially we're looking for a solution which looks like an oscillatory function outside the well $x < 0$ and $x > a$, but a decreasing exponential inside the potential barrier.

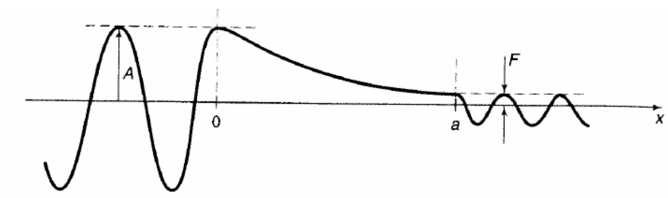

Figure 3: A sketch of the behavior of the wave function for tunneling through a broad and/or high barrier. Image from D. Griffiths.

Show then (argue this - no need for a precise mathematical calculation) that the relative amplitudes of the transmitted and incident waves are given by essentially the total decrease of the exponential over the barrier region, i.e.

$$\frac{|F|}{|A|} \sim e^{-1/\hbar \int_0^a |p(x)|dx},$$

so that

$$T \simeq = e^{-2\lambda} \qquad \text{where} \qquad \lambda = \frac{1}{\hbar} \int_0^a |p(x)|dx.$$

6 Putting it all Together - Stage 6

Gamow used the WKB approximation (though he didn't call it that) to finally derive the decay constant for the alpha decay of Uranium. Taking the nuclear strong force potential as a simple well coupled with a Coulombic electromagnetic potential, Gamow sketched out the potential of Uranium as follows.

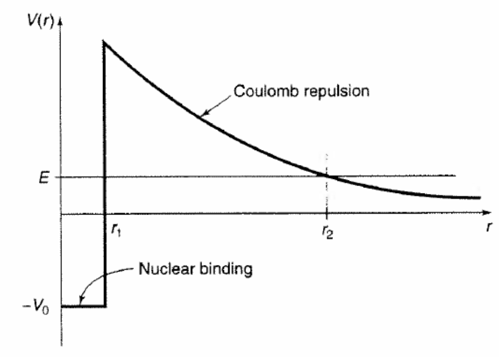

Figure 4: The potential of Uranium. E is the energy of the alpha particle. r_1 is the radius of the nucleus (and the size of the finite square well potential which models the nuclear strong force). r_2 is the outer turning point for an alpha particle of energy E. Image from D. Griffiths.

18. Show that the outer turning point r_2 is given by (using cgs units)

$$r_2 = \frac{2Ze^2}{E},$$

where Z is the proton number of Uranium, e is the charge of the electron in stat-Coulombs, and E is the energy of the alpha particle.

19. Next write down the integral for λ and show that it is given by

$$\lambda = \frac{1}{\hbar} \int_{r_1}^{r_2} \sqrt{U(r) - E} dr.$$

20. Show that the integral for λ can be re-written as

$$\lambda = \frac{\sqrt{2mE}}{\hbar}\int_{r_1}^{r_2}\sqrt{\frac{r_2}{r} - 1}dr.$$

At this point Gamow had trouble performing this integral, and in his autobiography reminisces:

> "I went to see my old friend N. Kotshchin, a Russian mathematician who was also spending that summer in Gottingen. He didn't believe me when I said I could not take the integral, saying he would give a failing grade to any student who couldn't do such an elementary task ... Later, when the paper appeared, he wrote to me that he had become a laughingstock among his colleagues, who had learned what kind of highbrow mathematical help he had given me."

21. Perform the integral using a u-substitution (let $r = r_2\sin^2(u)$). Since $r_1 \ll r_2$, use the small angle approximation for $\sin(r_1/r_2)$.

22. Finally show that

$$\lambda = K_1\frac{Z}{\sqrt{E}} - K_2\sqrt{Zr_1},$$

and determine the values of the constants K_1 and K_2. The units of K_1 should be $\mathrm{MeV}^{1/2}$ and K_2 should have units of $\mathrm{fm}^{-1/2}$. Here's where our practice using combinations of mc^2, α, and $\hbar c$ to easily determine numerical values of expressions will come in handy.

23. Determine r_1 by using the typical size of a nucleus, i.e. $R = 1.23$ fm $A^{1/3}$. What's typically done in the literature is to fudge r_1 by about half the width of an alpha particle, i.e. r_1 has the value you calculated here plus half of the size of a helium nucleus.

24. Now that we have the tunneling probability (and the decay constant), we need to figure out how to calculate the life-time of the Uranium atom. Here's where we bring in a semi-classical approximation. Suppose the alpha particle is moving inside the Uranium nucleus with some speed v_α. It takes the alpha particle roughly $2r_1/v_\alpha$ seconds to traverse the Uranium nucleus interior, after which it collides with a wall and is either reflected or transmitted. Each encounter has an extremely small chance that it ends

up with the alpha particle tunneling, $e^{-2\lambda}$. But, these collisions occur many times per second and one mole of a substance contains approximately 10^{23} nuclei, so on average a few alpha particles will succeed despite the long odds. Show therefore that the half-life of Uranium is given by

$$t_{1/2} = \frac{2\ln(2) r_1}{v_\alpha} e^{-2\lambda}.$$

25. To determine the value of v_α in our formula above, use the fact that a the depth of a typical nuclear well is approximately 35 MeV.

26. Finally, calculate the half-life of Uranium-238 given that it emits an alpha particle of 4.2 MeV. Compare this to the measured value of 1.41×10^{17} seconds.

7 The Geiger-Nuttall Law - Stage 7

The Geiger-Nuttall Law was an empirical relationship first noticed by Hans Geiger and John Mitchell Nuttall in 1911 between the decay constant for alpha-emitters like Uranium and Polonium and the energy of the emitted alpha particles. In particular, they noticed that half-lives are exponentially dependent on the emitted alpha energies. What this means is that very small changes in the emitted alpha energy can lead to very large changes in the half-life. For example, isotopes of Uranium and Thorium that emitted alpha particles with energies around 4 MeV tended to have half-lives on the order of billions of years, while those that emitted alpha particles with energies on the order of six MeV tended to have half-lives on the order of hours.

The modern day Geiger-Nuttall Law can be written as

$$\ln(\lambda_{decay}) = -a_1 \frac{Z}{\sqrt{E}} + a_2,$$

where λ_{decay} is the decay constant ($\lambda_{decay} = \ln(2)/t_{1/2}$), a_1 and a_2 are constants, Z is the atomic number, and E is the energy of the emitted alpha particle.

27. Derive the Geiger-Nuttall Law from your work in the previous stages. Find the constants a_1 and a_2.

28. For the following table (data courtesy Leon van Dommelen), use the Geiger-Nuttall law that you have derived to compute the half-life of the following alpha-emitters. The experimental values are included. How accurate are the half-lives and decay constants that you compute? Can you improve your calculations by changing some of the assumptions you made about various input parameters like v_α, r_1, etc.?

Parent Nucleus	E (MeV)	$t_{1/2}$ Experimental	λ_{decay} (s^{-1}) Experimental
Th^{232}	4.05	1.41×10^{10} yr	1.57×10^{-18}
Th^{228}	5.52	1.9 yr	1.16×10^{-8}
Rn^{222}	5.59	3.83 days	2.10×10^{-6}
Po^{218}	6.12	3.05 min	3.78×10^{-3}
Po^{216}	6.89	0.16 sec	4.33
Po^{214}	7.83	1.5×10^{-4} sec	4.23×10^{3}
Po^{212}	8.95	3.0×10^{-7} sec	2.31×10^{6}

29. Plot $\ln(t_{1/2})$ vs. $1/\sqrt{E}$ (where $t_{1/2}$ is measured in years and E in MeV) for the nuclei above. Notice the beautiful linear relationship.